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Abstract — **It is shown how to decompose any Binary Input Symmetric Memoryless Channel (BISMC) into BSCs. We explain why BEC and BSC are the two extreme cases of any BISMC with respect to the Mutual Information Profile (MIP). Finally, the paper points out useful applications for MIP.**

I. Introduction

By introducing the Mutual Information Profile (MIP) there is a new, alternative way of completely characterizing a BISMC which is equivalent to the traditional one given by the channels transition probabilities but describes the behaviour of a channel in a more evident way and thus makes some theorems easier to be proofed. Thinking in mutual information profiles is especially interesting when talking about information combining.

II. Decomposing BISMCs

It is shown that any Binary Input Symmetric Memoryless Channel (BISMC) can be decomposed into BSCs (see also [1]). A subchannel of a BISMC which is not further decomposable has an output alphabet with one or two output symbols. This will be proofed by contradiction where we assume that the subchannel a has more than two different output symbols. As the subchannel is strongly symmetric, the transition matrix has only two probabilities p_1, p_2 and all of its columns are permutations of each other (note that there are only two different permutations). If $p_1 = p_2$, the transition matrix of the considered subchannel can be further decomposed into single columns, each of them corresponding to a BSC with error probability 1/2. If $p_1 \neq p_2$ we group the two different columns to submatrices and decompose the BISMC further into channels corresponding to these matrices (decomposition into BSCs). Decomposition of subchannel a is a contradiction to our assumption (q. e. d.).

Along with the decomposition into BSCs we introduce the MIP $w_I(i)$, which is the probability density function of mutual information. Note that the average mutual information over the BISMC I is given by the first moment of the MIP.

An AWGN channel with binary input ± 1 is a BISMC and can be decomposed using $a = |y|$ as a subchannel-indicator (y is the continuous output signal). The pdf of a is

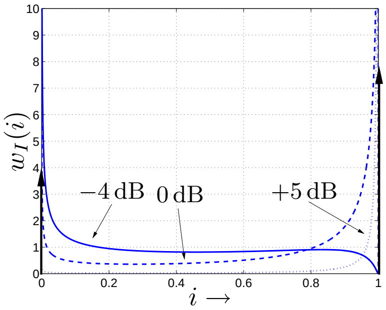
$$f_A(a) = \frac{1}{\sqrt{2\pi}\sigma_n}\left(\mathrm{e}^{-\frac{(a+1)^2}{2\sigma_n^2}} + \mathrm{e}^{-\frac{(a-1)^2}{2\sigma_n^2}}\right), \tag{1}$$

where $\sigma_n^2 = \frac{1}{2}\cdot\frac{N_0}{E_s}$ is the noise power. The mutual information $I(a)$ of a subchannel and the MIP $w_I(i)$ are given by

$$I(a) = 1 - \mathrm{h}\left(\frac{1}{1+\mathrm{e}^{2a/\sigma_n^2}}\right) \tag{2}$$

$$w_I(i) = \left.\frac{f_A(a)}{\mathrm{d}I(a)/\mathrm{d}a}\right|_{a=I^{-1}(i)}, \quad \frac{\mathrm{d}I(a)}{\mathrm{d}a} = \frac{2\cdot\mathrm{e}^{2a}\cdot\log_2(\mathrm{e}^{2a})}{(1+\mathrm{e}^{2a})^2} \tag{3}$$

The figure shows MIPs for an AWGN channel at different values $10\log_{10}(E_s/N_0)$. For fixed capacity BEC and BSC



are the two extreme cases w. r. t. the MIP, having maximum or minimum (zero) second moment, respectively. Thus, BEC and BSC are extreme cases for all bounds derived by means of Jensen's inequalities, e. g. information combining [2, 3]. Using these extreme cases, a lot of bounds like Fano's and Hellman-Raviv-bound can be derived in a very illustrative way. For high SNR, the AWGN channel can be approximated very accurately by a BEC with equal capacity (see also the plot for $10\log_{10}(E_s/N_0) = 5\,\mathrm{dB}$, well approximated by two Dirac δ-functions for $i = 0$ and $i = 1$).

III. Applications

High-rate codes are used in high signal to noise ratio scenarios. As for high SNR an AWGN channel can be well approximated by a BEC, high-rate codes can be designed for BECs, e. g. by very efficient density evolution and used for AWGN channels, like shown in the following table:

Rate	0.2	0.5	0.99
Rel. AWGN Cap. Gap	0.00582	0.00357	0.00078

In [4] another case has been studied: Nonoptimized codes perform equally well for AWGN and BEC at any SNR.

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